Counterparty risk externality: Centralized versus over-the-counter markets [*]

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Abstract

We study financial markets where agents share risks, but have incentives to default and their financial positions might not be transparent, that is, might not be mutually observable. We show that a lack of position transparency results in a *counterparty risk externality*, that manifests itself in the form of excess "leverage," in that parties take on short positions that lead to levels of default risk that are higher than Pareto-efficient ones. This externality is absent when trading is organized via a centralized clearing mechanism that provides transparency of trade positions. Collateral requirements and especially subordination of non-transparent positions in bankruptcy can ameliorate the counterparty risk externality in market settings such as over-the-counter (OTC) markets which feature a lack of position transparency.

1   Introduction and motivation

An important risk that needs to be evaluated at the time of financial contracting is the risk that a counterparty will not fulfill its future obligations. This counterparty risk is difficult to evaluate because the exposure of the counterparty to various risks is generally not public information. Contractual terms such as prices and collateral that affect a trade can be tailored to mitigate counterparty risk, but the extent to which this can be achieved, and how efficiently so, depends in general on how contracts are traded.

Consider a market in which each party trades with another, subject to a bankruptcy code that determines how counterparty defaults will be resolved.[1] A key feature of many such markets, for instance of OTC markets, is their *opacity.* In particular, even within a set of specific contracts, for example, credit default swaps (CDS), no trading party has full knowledge of positions of others.[2] We show theoretically that such opacity of exposures, or the lack of position transparency, leads to an important risk spillover – *a counterparty risk externality*[3] – that, in turn, leads to excessive "leverage" in the form of short positions that collect premium upfront but default ex post. Such excessive leverage results in inefficient levels of risk-sharing and in deadweight costs of bankruptcy.

Counterparty risk externality is the effect that the default risk on one contract will be increased if the counterparty agrees to any contract with another agent which increases the probability that the counterparty will be unable to perform on the first one. This is typically the case, for instance, if all else equal (that is, without any increase in its endowments), a counterparty that has sold insurance sells more insurance to other agents. Put simply, the default risk on one deal depends on *what else is being done.* The intuition for our result concerning the counterparty risk is that an externality arises when portfolio positions are not transparent. In this case, counterparties cannot charge price schedules that effectively penalize the creation of ineffi-

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[1]The contract may adhere to a uniformly applicable corporate bankruptcy code, or when the contract is exempt from the code, the bankruptcy outcome may be specified in the contract.

[2]CDS contracts do effectively reflect counterparty risk as collateral arrangements embedded in the contracts depend on counterparty risk (credit rating, for example). Nevertheless, such collateral arrangements are bilateral and do not depend on the aggregate position of the counterparty.

[3]The term "counterparty risk externality" is as employed by Acharya and Engle (2009). A part of the discussion below, especially related to A.I.G. is also based on that article.

cient levels of counterparty risk. This makes it likely that excessively large short positions will be built by some institutions without being discouraged to do so by pricing or risk controls tied to their overall positions.

For example, in September 2008, it became known that A.I.G.'s liquidity position was inadequate given that it had written credit default swaps (bespoke CDS) for many investors guaranteeing protection against default on mortgage-backed products. Each investor realized that the value of A.I.G.'s protection was dramatically reduced on its individual guarantee. The counterparty risks were so widespread globally that a default would probably have spurred many other defaults, generating a downward spiral. The A.I.G. example illustrates the cost that large non-transparent exposures can impose on the system when a large institution defaults on its obligations. We argue that the opacity of the OTC markets in which these credit derivatives traded may in part have been responsible for allowing the build-up of such large exposures in the first place.Indeed, a number of financial innovations in fixed income, foreign exchange, and credit markets have traded until now in non-transparent markets, the (gross) global notional outstanding of such derivatives being close to $500 trillion in December 2009, as per the Global Financial Stability Report of the IMF (April 2010).

As a way to address the inefficiencies caused by opacity of markets, recent regulatory reforms have proposed a centralized clearing mechanism that registers all trades in financial markets and then serves as a data repository providing transparency of these trades. We show formally that when trading is organized in the form of such a centralized clearing mechanism, position transparency can enable market participants to condition contract terms for each counterparty based on its overall positions. Such conditioning is sufficient to get that party to internalize the counterparty risk externality of its trades and achieve the efficient risk-sharing outcome. In other words, the moral hazard that a party wants to take on excessive leverage through short positions – collect premiums today and default tomorrow – is counteracted by the fact that they face a steeper price schedule by so doing.

1.1 Model and results

We formalize these intuitions in a competitive two-period general equilibrium economy which allows for default.[4] There is a single financial asset, which can be interpreted as a contingent claim on future states of the world. Agents can take long or short positions in the asset. Trades are backed by agents' endowments. When an agent has short positions that cannot be met by the pledgeable fraction of endowment, there is default. Default results in deadweight costs which are borne by the short position and are increasing in the size of short positions, e.g., due to a greater number of parties to deal with in a bankruptcy proceeding. Such costs may arise also due to loss of customers or franchise value in fully dynamic setups. We do not model the structure of bankruptcy costs but simply postulate their pecuniary equivalent in reduced form.

The possibility of default (the option to exercise limited liability, to be precise) implies that long and short positions do not necessarily yield the same payoff and indeed that there might be counterparty risk in trading. We assume a natural bankruptcy rule that illustrates why counterparty risk potentially arises in such a setting. In particular, in any given state of the world, the payoff to long positions is determined pro-rata based on delivery from short positions. This rationing of payments implies that each trade imposes a payoff externality on other trades. This spillover is precisely what we refer to as a *counterparty risk externality.*

In this setup, we consider a centralized clearing mechanism with transparency, a market structure which guarantees that all trades are observable and agents can set pricing schedules that are conditional on this knowledge. We contrast this market structure with another where trades are not mutually observed and thus pricing schedules faced by agents are not conditional on their other trades (even though they might be conditioned on public information about their type, e.g., their level of endowment).

In this context, we first show that *competitive equilibria in economies with a transparent centralized clearing mechanism or a centralized exchange are constrained Pareto efficient.* This is true even allowing for market incompleteness so that the result is not simply a consequence of welfare theorems in case of complete markets. Our second result is that *competitive equilibria in economies with non-transparent portfolio positions are robustly constrained*

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[4]See Geanakoplos (1997), Geanakoplos and Zame (1998), and Dubey, Geanakoplos and Shubik (2005) for models of default in general equilibrium.

inefficient.

Intuitively, as long as there is a "risk premium" on the underlying contract (e.g., because the risk being insured in the contract is aggregate in nature) and the costs of defaulting are not excessively large, the short position (the insurer) perceives a benefit from collecting premiums upfront and defaulting ex post. We interpret this outcome as characterizing excessive "leverage."Formally, we capture the resulting inefficiency in the form of deadweight costs of bankruptcy.[5]

We also show that, in general, the counterparty risk externality is internalized only if the prices an agent faces for shorting an asset depend on her portfolio position as well as those of her direct counterparties, those of the counterparties of the counterparties, etc. A price mechanism which only depends on the positions of the agent shorting the asset guarantees efficiency only if agents never hold both long and short positions on the same financial asset. Since provision of such a high level of position transparency may be too costly or infeasible in some market settings, we consider in extensions the role of alternative regulatory mechanisms in addressing the counterparty risk externality. More specifically, we analyze the welfare effects of restricting trades to a competitive centralized exchange, requiring bilateral collateral arrangements, and subordinating non-transparent positions in bankruptcy relative to centrally cleared ones.

1.2   Related Literature

The literature on insurance provision through financial contracts (e.g., Duffee and Zhou, 2001, Acharya and Johnson, 2007, Parlour and Rajan, 2001) has largely focused on moral hazard on part of the insured due to lack of perfect information about the insured's characteristics. In contrast, our paper is concerned with moral hazard on part of the insurer, and how non-transparent markets contribute to it. Some of these aspects feature in the recent work of

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[5]More generally, the inefficiency could manifest as excessive systemic risk due to spillover on to other counterparties. The inefficiency could also translate into a production inefficiency: in the market for insurance on economy-wide mortgage defaults, in equilibrium, the insurer would take on large and inadequately-collateralized short-selling (of protection) on pools of mortgages and the insured lenders would feed the excessive creation of the housing stock backing such mortgages. This may be a partial explanation of the role played by credit default swaps, sold in large quantities by A.I.G. on corporate loan and mortgage pools, in fueling the credit boom preceding the crisis of 2007-09.

Thompson (2010) and Leitner (2009).[6]

Thompson (2010) considers how the possibility of default on part of the insurer provides incentives to the insured parties to reveal information about their type. That is, Thompson's focus is on the insurer moral hazard problem from the asset side, whereas our focus is on the liability side in that we examine how insured's incentives to hedge impose an externality on other creditors of the insurer. In Thompson's model, the optimal intervention features the planner imposing risk controls on the insurer. In contrast, transparency of positions in our model can enable the market to address the market failure. Our result is most closely related to Leitner (2009)'s result that a clearinghouse-style mechanism, by allowing each party to declare its trades and revealing publicly those that hit pre-specified position limits, can prevent agents from promising the same asset to multiple counterparties and then defaulting. Leitner's focus is on inducing the revelation of hidden trades by agents, whereas our focus is on enabling agents to write exclusive contracts that enable agents to internalize the counterparty risk externality.

Duffie and Zhu (2009) and Stephens and Thompson (2011) also discuss the relative benefits of centralized markets. In Duffie and Zhu (2009), netting across a large number of products is required for a centralized counterparty to reduce counterparty risk with respect to a non-transparent market setting. Our results on the other hand hold independently of institutional details regarding netting. In our context, in fact, the primary role of a centralized clearing mechanism (or of a centralized counterparty) is not necessarily to directly reduce or eliminate counterparty risk but to improve its price by aggregating information on trades. In Stephens and Thompson (2011) a centralized counterparty is associated with a mutualization of the counterparty risk. Risks are pooled by the centralized trading agency. As a consequence, there are adverse incentive effects when insurers are of different risk types. In our model, the role of a centralized counterparty is to provide position

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[6]The bilateral nature of contracts in non-transparent markets like OTC has been stressed in the recent literature on the subject. Duffie, Garleanu and Pedersen (2005, 2007) focus on search frictions, dynamic bargaining and valuation in OTC markets; Caballero and Simsek (2009) analyze the role of complexity introduced by bilateral connections and their role in causing financial panics and crises; and, Golosov, Lorenzoni, and Tsyvinski (2009) examine what kind of bilateral contracts will get formed when agents have private information about their endowment shocks. Our paper, while focused on non-transparent markets like OTC, is fundamentally different in that it is concerned primarily with issues of opacity and resulting inefficiencies.

transparency to the market, without providing any mutualization of risk. However, allowing for mutualization of risk would have similar adverse effects in our model too.

Counterparty risk and moral hazard are induced endogenously in our model by the opacity of agents' positions and their strategic default decisions. This is different, for instance, from other existing formalizations of counterparty risk which rely, as for instance do Stephens and Thompson (2011), on unobservable and heterogeneous default risk characteristics across firms rather than on non-transparent financial positions. In the terminology of existing literature, we compare competitive equilibria in *exclusive contractual environments* to competitive equilibria in *non-exclusive contractual environments*. Exclusive contractual environments are by definition those in which one party in a contractual relationship can constrain all of the counterparty's trades with third parties, typically at an optimal contract by implementing position limits, so that counterparty risks do not affect economic efficiency.[7] Bizer and DeMarzo (1992) is an early paper in finance which exploits the distinction between exclusive and non-exclusive contractual environments to study sequential debt accumulation in a model of banking. More recent applications of these concepts include Parlour and Rajan (2001) in a model of credit card loans, Bisin and Rampini (2006) in a model of bankruptcy, and Bisin and Rampini (2006b) in a model of public policy when government lack commitment.

Finally, we focused on symmetric information about states of the world in our analysis. However, there could be adverse selection, e.g., in the form of unobservable probability distributions over S, the uncertain state at date 1. Modeling adverse selection in our setup would require combining features of Rothschild and Stiglitz (1976) and Akerlof (1970).[8] We conjecture that there would be separating equilibria in the economy with centralized clearing, and excessive *lemons* trading (in the form of risky short positions) in the case of non-transparent markets. In turn, we conjecture that the inefficiency of non-

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[7]The distinction between exclusive and non-exclusive contracts is central in the theory of competitive economies with moral hazard; see e.g., Bisin and Gottardi (1999), Bisin and Guaitoli (2004), Bisin, Geanakoplos, Gottardi, Minelli and Polemarchakis (2001), and Dubey, Geanakoplos, and Shubik (2005). In the context of principal agent models, see the early work of Arnott and Stiglitz (1993), Hellwig (1983), and Eaton, Gersovitz, and Stiglitz (1984).

[8]Santos and Scheinkman (2001) have adverse selection as well in their model of competition of exchanges.

transparent markets will be exacerbated in a setting with adverse selection.

The remainder of the paper is structured as follows. Section 2 provides a simple example of the counterparty risk externality in non-transparent markets. Section 3 presents the general model, the various trading structures (non-transparent and centralized clearing with transparency), and the welfare analysis of competitive equilibrium under these structures.Section 4 discusses the extensions of the model. Section 5 concludes and discusses the policy implications. The Appendix contains proofs.

2 Counterparty risk externality: An example

Consider a two-period $(t = 0, 1)$ competitive economy with three types of agents $(i = 1, 2, 3)$. There are two states of the world at $t = 1$, denoted by Good (G) and Bad (B). The probabilities of these states are p and $(1 - p)$, respectively. Agents' endowments in the two states are denoted as $w^i(s)$, $i = 1, 2, 3$, and $s = G, B$. Their initial endowments are denoted w_0^i. We assume that initial endowments are large enough that there are no default considerations at $t = 0$. For simplicity, we also assume that

$$w^1(G) > w^2(G) > w^3(G) = 0 \text{ and } w^1(B) = w^2(B) = 0 < w^3(B).$$

In other words, agents of type 1 and type 2 have endowment in the good state of the economy, but none in the bad state; agents of type 3 are endowed in the bad state but not in the good state.

Agents of each type have a mean-variance utility function:

$$E[u(x_0, x(s)] = x_0 + E(x(s)) - \frac{\gamma}{2} var(x(s)),$$

where x_0 is consumption at $t = 0$, and $x(s)$ is consumption at $t = 1$ in state s.

The only traded contract is an "insurance" that resembles a put option on the bad state of the economy. The contractual payoff of the contract is $R(G) = 0$ and $R(B) > 0$. For simplicity, we will refer to $R(B)$ simply as R. Importantly, the economy will allow for default so that the actual payoff on the contract in the bad state may be less than R. The insurance contract must be paid for at $t = 0$ and we denote its price as q.

To highlight our main point, we consider agents 1 and 2 purchasing insurance contract from agents 3.[9] We denote the long positions of agents 1 and 2 as $z^i \geq 0$, $i = 1, 2$, and the short position of agents 3 as $z^3 \geq 0$. Note that the only agents that can default given our assumptions are agents 3. In case they default, they suffer a linear non-pecuniary penalty as a function of the positions defaulted upon, whose pecuniary equivalent in the bad state is given by ϵz^3.

2.1 Non-transparent markets

We consider trading in non-transparent markets: agents do not observe the size of the trades put on by other agents and hence prices cannot be conditioned on these. In other words, all agents take the price per unit of insurance as a given constant (and not a schedule depending on total insurance sold by agents 3 in the economy). Agents are fully rational, however, and anticipate correctly the likelihood of default, and its consequent effect on the realized payoff on the insurance contract (R^+) relative to the promised payoff (R), with $R^+ \leq R$.

Equilibrium in the economy is characterized by the trading positions, the payoff on the insurance contract (involving the possibility of default), and the cost of insurance, denoted as (z^1, z^2, z^3, R^+, q), such that:

1. Each agent maximizes its expected utility by choosing its trade positions (as we describe below);

2. Market for insurance clears: $z^3 = z^1 + z^2$; and,

3. In case of default, (we assume that) agents 3's total endowment is shared pro-rata between the long positions of agents 1 and 2: :

$$R^+ = \begin{cases} \frac{w^3(B)}{z^1 + z^2} & \text{if } 1_D = 1 \\ R & \text{else} \end{cases}$$

   where 1_D is an indicator variable which takes on the value of one if there is default ($R^+ < R$) and zero otherwise.

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[9]It would suffice to consider just two types of agents since each agent type is a continuum of identical agents. Nevertheless, for sake of a clearer exposition of the counterparty risk externality, we consider three types of agents.

Now, consider agent $i = 1, 2$'s maximization problem:

$$max_{z^i} \ w_0^i - z^i q + p w^i(G) + (1-p)R^+ z^i - \frac{\gamma}{2} var(x^1(s)),$$

where

$$var(x^i(s)) = p(1-p)[w^i(G) - R^+ z^i]^2.$$

Then, the first-order condition for agent $i = 1, 2$ implies that:

$$z^i(R^+, q) = \frac{1}{R^+} \left[w^i(G) - \frac{(q - (1-p)R^+)}{\gamma p(1-p)R^+} \right]. \tag{1}$$

In other words, all else equal, agents 1 and 2 purchase more insurance if they have greater endowment in the good state and less so if the cost of insurance rises. The crucial observation is that even though the payoff R^+ is affected by each agent's long position in equilibrium, agents are competitive and do not internalize this effect. This is the source of counterparty risk externality in the model. In a GE model without default, R^+ is guaranteed to be R so that the externality would not arise.[10]

Next, we will show that agents of type 3 have incentives to default in state B whenever the parameter governing the deadweight cost of default, ε, is not too high. To clarify agent 3's choice with regard to default, consider first the case in which it cannot default. In this case, agent 3 would sell insurance in the amount

$$z_{ND}^3 = \frac{1}{R} \left[w^3(B) + \frac{(q - (1-p)R)}{\gamma p(1-p)R} \right]. \tag{2}$$

In the limit where there are no default costs, that is, $\epsilon = 0$, agent 3 with position z_{ND}^3 will not default in equilibrium only if

$$w^3(B) \geq R z_{ND}^3,$$

which turns out to be equivalent to requiring that $q \leq (1-p)R$. This condition has the intuitive interpretation that the insurer has incentives not

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[10]The externality is akin to a "run" in the context of financial intermediaries that sell claims with sequential service contracts such as demandable deposits: If the payoff on claim is not anticipated to be the promised amount, each depositor runs in order to obtain its desired payoff, but such runs can lead to externality on other creditors who have less to claim their desired payoffs from.

to default ex post only if the price of insurance is smaller than or equal to the expected payoff on the insurance, or in other words, that there is no "risk premium" in the insurance price. This will, however, not hold in equilibrium in general, whenever the insurance is against a risk that is aggregate in nature and cannot be fully diversified away, e.g., if $w^1(G) + w^2(G) > w^3(B)$.[11]

Consider instead the problem of agent 3, the insurer, conditional on defaulting:

$$max_{z^3} \; w_0^3 + z^3 q - (1-p)\epsilon z^3 - \frac{\gamma}{2}p(1-p)(\epsilon z^3)^2.$$

Clearly, in this case the insurer pledges the entire endowment in the bad state at $t = 1$ in order to collect as much insurance premium as possible at $t = 0$. From the first-order condition, we obtain that

$$z^3 = \frac{q - (1-p)\epsilon}{\gamma p(1-p)\epsilon^2}. \tag{3}$$

Thus, the lower the cost of default ϵ and greater the price of insurance q, the greater is the quantity of insurance supplied by the insurers.

To develop more intuition about the equilibrium, let "risk premium" be:

$$\Delta p = \frac{q}{R^+} - (1-p); \tag{4}$$

that is, the difference between the "risk-neutral" probability of state B and its actual or statistical probability. Then, solving for equilibrium in Δp and R^+ yields:

$$\Delta p \;\; = \;\; \frac{1}{2}\gamma p(1-p)\left[w^1(G) + w^2(G) - w^3(B)\right], \tag{5}$$

$$R^+ \;\; = \;\; \frac{(1-p)\varepsilon + \sqrt{(1-p)^2\varepsilon^2 + 4w^3(B)\gamma p(1-p)\left[\Delta p + (1-p)\right]\varepsilon^2}}{2\left[\Delta p + (1-p)\right]} \tag{6}$$

In other words, there is a risk premium whenever agents are risk-averse ($\gamma > 0$), there is risk ($0 < p < 1$), and this risk cannot be diversified away across agents ($w^1(G) + w^2(G) > w^3(B)$). Furthermore, R^+ is increasing in ε: the higher the bankruptcy costs, the lower is the equilibrium default rate on the contract. It follows then that the contract price $q = \left[\Delta p + (1-p)\right] R^+$ is also increasing in ε. In turn, there is default in equilibrium ($R^+ < R$) if and only

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[11]Note that this notion of risk premium is due to the aggregate nature of a risk rather than a difference in the risk-aversions of buyers and sellers.

if bankruptcy costs are sufficiently small (ε smaller than some threshold level $\bar{\varepsilon}$).

Suppose now that z^3 were observable and the planner could impose a pricing rule $q(z^3)$ to be (i) $[\Delta p + (1-p)]R$, whenever there is no default (i.e., $w^3(B) \geq Rz^3_{ND}$); and, (ii) $[\Delta p + (1-p)]\frac{w^3(B)}{z^3}$, when there is default. Then, substituting the default-region price map into agent 3's maximization (2.1), we see that there is no longer an incentive to default: the proceeds from selling contracts $q(z^3)z^3$ are equal to $[\Delta p + (1-p)]w^3(B)$, which is invariant to z^3, whereas the insurer suffers deadweight costs of default from selling z^3 to be high enough to be in this default region. Agents 1 and 2 continue to purchase insurance in a competitive manner under this price map (since z^3 is taken as given by each of these agents). However, since agent 3 receives no benefit and only default costs for selling insurance beyond $z^3 > z^3_{ND}$, there would be no supply of insurance in equilibrium beyond z^3_{ND} and the counterparty risk externality eliminated.[12]

To better illustrate the most important properties of equilibrium we now parametrize the above economy with $w^1(G) = 10$, $w^2(G) = 5$, and $w^3(B) = 10$ so that state B is aggregate risky in nature. We set $\gamma = 1$, $p = 0.9$ and vary ϵ in the range $[0.1, 1.0]$ (a subset of the entire possible range $\epsilon > 0$). Figures 1, 2 and 3 plot respectively the equilibrium quantity of insurance sold (z^3), its realized payoff (R^+), and its price (q), all as a function of ϵ, the proportional deadweight cost of default.

There is a critical value of ϵ below which defaults take place and this value is around 0.548. Above this value, there is no default. Interestingly, for all ϵ smaller than this threshold value, the equilibrium is effectively the same as far as risk-sharing is concerned: agents of type 3 transfer all their endowment in the bad state at $t = 1$ to agents 1 and 2. To be precise, the equilibrium utilities (relative to $t = 0$ endowments) are $(U^1, U^2, U^3) = (-1.97, -0.84, 1.35)$ regardless of ϵ in the default range. However, this is not true of the equilibrium quantity of insurance contracts sold and the unit price of insurance.

For example, when $\epsilon = 0.5$, the quantities traded are $(z^1, z^2) = (8.22, 2.74)$ with $z^3 = z^1 + z^2$; there is 9% default on the contract ($R^+ = 0.91$); and, insurance price is $q = 0.30$. In turn, the risk premium Δp equals 0.23.

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[12]Alternatively, and within this specific example, the planner can also enforce a "position limit" that restricts agents of type 3 from selling a quantity of insurance z^3 that cannot be met by their endowment in the bad state $w^3(B)$.

In contrast, with $\epsilon = 0.01$, the quantities traded become much larger: $(z^1, z^2) = (410.95, 136.98)$; there is 98% default on the contract ($R^+ = 0.02$); and, insurance price is much lower at $q = 0.0067$.

To summarize, as the default incentives for agents of type 3 become stronger, there is greater quantity of insurance sold, greater default, and greater deadweight costs suffered by these agents. In turn, the equilibrium insurance price is smaller too. Since the payoff on the contract is rationally anticipated by those purchasing insurance to be smaller: the quality of insurance has gone down given the insurer's default risk. Interestingly, there is no effect of default risk on the risk premium, which is constant and is given by equation (5).

2.2 Inefficiency of non-transparent markets

The inefficiency of equilibrium in the example above when $\varepsilon < 0.548$ stems from excessive deadweight costs of agent 3's bankruptcy. This can be seen in Figure 4 which plots the sum of utilities of all three agents and also separately of agents of type 3. Agents 1 and 2 enjoy the same equilibrium utility as ϵ varies. However, for $\epsilon < 0.548$, default leads to deadweight costs borne by agents of type 3 and their equilibrium utility is substantially lower compared to the case where $\epsilon \geq 0.548$. The result of counterparty risk externality is that there is too much demand for insurance in equilibrium, which gives insurers the incentive to default ex post, for which they pay ex ante.

As explained above, in the example the planner can improve upon the non-tranparent markets case when ϵ is smaller than 0.548 by subjecting the insurer to the price map (q as a function of z^3) in Figure 3. While in this example, it is efficient for insurance to be fully collateralized so that any default is ruled out in equilibrium, this is in general not true as some default may be efficient in equilibrium to produce Pareto-improving state-contingency in contract payoffs when markets are incomplete.[13] Further, the example abstracts from the fact that in practice agents (institutions) often take long and short sides of contracts with different counterparties, so that in case of bankruptcy, how these contracts are netted becomes important. The example also makes it clear that there is an inherent tension between the notion

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[13]What is however true, and we show below, is that non-transparent markets always feature (weakly) greater likelihood of default in equilibrium compared to its (Pareto) efficient level.

of competitive equilibrium and prices or position limits that are conditioned on aggregates of individual choices.

Importantly, the "right" informativeness of prices provides incentives on and off equilibrium (the region of non-linear pricing is never reached as the price collapses to zero beyond the "position limit"). Given the information requirements of such prices, which may be onerous in some market settings, and given the right conditionality required of such prices, alternative ways to ameliorate the counterparty risk externality may be desirable. In particular, one might think that instruments controlling the cost of default (e.g., collateral) might be enough, or that incentives to guarantee that trades occur only in transparent markets (e.g., seniority in bankruptcy of transparent positions relative to non-transparent ones) may suffice. It turns out, however, that such alternative mechanisms ameliorate the counterparty risk externality, but do not eliminate it. These important issues can be fully understood only by setting up the general formulation below.

3   The model and results

The economy in our general model has two dates as before ($t = 0, 1$) and is populated by $i = 1, ..., I$ types of agents. Let x_0^i be consumption of agent i at time 0. Let $s = 1, ..., S$ denote the states of uncertainty in the economy, which are realized at time 1. State s occurs with probability p_s, and $\sum_s p_s = 1$.

Let x_1^i be agent i's consumption at time 1, a random variable over the state space S: $x_1^i(s)$, for $s \in S$. Let w_0^i be the endowment of agent i at time 0; and $w_1^i(s)$ her endowment at time 1 in state s . The utility of agent i over consumption is time separable with component utility $u_0^i(x)$ and $u_1^i(x)$ (resp. at time 0 and at time 1). We assume $u_0^i(x)$, $u_1^i(x)$ are well-behaved for any $i \in I$: in particular, we require that $u_0^i(x)$ is strictly increasing and $u_1^i(x)$ is strictly concave. While we do not need to assume a von Neuman-Morgenstern representation of preferences at time 1, it is convenient to do so.[14] We assume no discounting just for notational simplicity.

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[14]The particular form of mean-variance preference assumption we assumed in the example in Section 2 does not satisfy a von Neumann-Morgenstern expected utility representation. A representation which would nest our mean variance assumption as well as an expected utility representation exists (see the **Note** in the Appendix); all results in the paper would go through in this case.

Financial markets and default We assume, for simplicity, that only one financial asset is traded in this economy, an asset whose payoff is an exogenous non-negative S-dimensional vector $R(s)$. We can imagine it representing a derivative contract, e.g., a credit default swap.

Agents selling the asset might default on their required payments. In particular, agent i's short positions are effectively backed by the pledgeable fraction α of her endowment at time 1 and by the payoff of the financial assets in her portfolio. In other words, in the event of default, creditors (counterparties holding long positions on the asset with the defaulting party) have recourse to her financial assets and only to a fraction $\alpha \in [0,1]$ of agent i's endowment $w_1^i(s)$. The non-pledgeable part of the endowment could represent the agent's human capital that is valuable to the firm in presence of the agent, but this value cannot be transferred to firm's creditors, e.g., because the agent leaves the firm in case of bankruptcy or switches to alternative employment.[15]

Other than the defaulting agent simply losing her pledgeable endowment to counterparties, default is assumed to have a direct deadweight cost that is proportional to the size of the position defaulted upon. Deadweight costs of default will serve the formally convenient purpose of providing a bound on short positions on the asset. Our results are qualitatively unaffected if deadweight costs of default are assumed to be proportional to the unpaid portion of short positions rather than total short positions.

A bankruptcy mechanism operates to distribute the cash flow delivered on the short positions pro-rata amongst the long positions. To be precise, consider an agent of type i shorting the asset. At equilibrium, the total repayment cash flow from an agent of type i is distributed pro-rata among the holders of long positions against counterparty i.[16]

Agents trade in competitive financial markets. Even though one single asset is traded ex ante, the asset pay-off ex post depends on the type of the agent shorting it, as that agent's default decision also depends on the type. Let z_+^{ij} be long positions of agents of type i sold by agents of type j.[17] Let

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[15]Our results are robust, as in the example, to setting $\alpha = 1$. We present analysis with general α, however, as some non-pledgeability of agents' endowments is natural.

[16]Given the competitive nature of the model, the bankruptcy mechanism pools all repayments of *all* agents of type i and redistributes them pro-rata to all their counterparties. This is without loss of generality, as we concentrate on symmetric equilibria.

[17]Note that the apex ij refers to the *types* of the agents engaged in the trade, not to their individual names. Trades are not literally bilateral, in the sense that markets are

$\mathbf{z}_+^i = \left(z_+^{ij}\right)_{j \in I} \in \Re_+^I$ denote the long portfolio vector of agents of type i (with $z_+^{ii} = 0$, by construction). Let $z_-^i \in \Re_+$ be the short position of agents of type i. All short positions are symmetric for the agents shorting the asset, independently of the counterparty, so that there is no need to index short positions of an agent by the counterparty. Then, in case of its default, agent i suffers a deadweight cost of default whose pecuniary equivalent is assumed to be εz_-^i, with $\varepsilon > 0$.

Default and payoffs on long and short positions An agent of type i with (long, short) portfolio position $(\mathbf{z}_+^i, z_-^i) \in \Re_+^{I+1}$ will default in period 1 in state s if and only if her income after her long positions on assets have paid off is smaller than the non-pledgeable fraction of her endowment net of the bankruptcy costs.[18] Since all long positions share pro-rata the payments from defaulting and non-defaulting short positions, the payoff in state s of the asset shorted by agent j depends on agent j's default decision which in turn depends on her portfolio position, $\left(\mathbf{z}_+^j, z_-^j\right)$, and on all other agents' portfolio positions, $\left(\mathbf{z}_+^i, z_-^i\right)$, for any $i \in I \backslash \{j\}$.

Let $(\mathbf{z}_+, \mathbf{z}_-) = \left(\mathbf{z}_+^i, z_-^i\right)_{i \in I} \in \Re_+^{I(I+1)}$. The payoff in state s of the asset shorted by agent j, for given portfolio positions $(\mathbf{z}_+, \mathbf{z}_-)$, is denoted $R^j(\mathbf{z}_+, \mathbf{z}_-; s)$. Agent j's income in state s if she does not default is then

$$Y_{ND}^j (\mathbf{z}_+, \mathbf{z}_-; s) = w_1^j(s) + \sum_i R^i(\mathbf{z}_+, \mathbf{z}_-; s) z_+^{ji} - R(s) z_-^j.$$

On the other hand, if agent j defaults, her income is

$$Y_D^j (\mathbf{z}_+, \mathbf{z}_-; s) = (1 - \alpha) w_1^j(s) - \varepsilon z_-^j.$$

The payoff in state s of the asset shorted by agent j, $R^j(\mathbf{z}_+, \mathbf{z}_-; s)$, is implicitly defined by the following fixed-point condition:

$$R^j(s)(\mathbf{z}_+, \mathbf{z}_-) = \begin{cases} \frac{\sum_i R^i(s)(\mathbf{z}_+, \mathbf{z}_-) z^{ji} + \alpha w_1^j(s)}{z_-^j} & \text{if } Y_{ND}^j (\mathbf{z}_+, \mathbf{z}_-; s) \leq Y_D^j (\mathbf{z}_+, \mathbf{z}_-; s) \\ R(s) & \text{if } Y_{ND}^j (\mathbf{z}_+, \mathbf{z}_-; s) \geq Y_D^j (\mathbf{z}_+, \mathbf{z}_-; s) \end{cases}$$

$$(7)$$

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competitive.

[18]In general we allow for an agent to maintain at the same time both short and long positions on the asset: z_-^i and $z_+^{ij} > 0$, for some $i \neq j$ (we adopt the convention $z_+^{ii} = 0$, for any i). In other words, we assume that the clearing mechanism does not necessarily include *netting*. We shall discuss netting later on in the section.

The fixed-point condition implicitly defines the payoff of a short position of the asset by agent j, $R^j(\mathbf{z}_+, \mathbf{z}_-; s)$ as a correspondence. It is a correspondence because the payoff of agent j when the condition for default in (7) holds with equality for any agent in the economy is not uniquely determined. It is easy to see in fact that $R^j(\mathbf{z}_+, \mathbf{z}_-; s)$ is well-behaved as a correspondence.

Lemma 1. *The map $R^j(\mathbf{z}_+, \mathbf{z}_-; s)$ defined by the fixed-point condition (7) is a non-empty-valued upper-hemi-continuous correspondence.*

At equilibrium, agents will rationally coordinate their expectations on a selection of $R^j(\mathbf{z}_+, \mathbf{z}_-; s)$. To simplify notation, in the paper we proceed as if the selection agents coordinate upon is known and (abusing notation) we denote it $R^j(\mathbf{z}_+, \mathbf{z}_-; s)$. In the Appendix we discuss these issues more in detail, including the issue of the convexification necessary for existence.

3.1   Non-transparent markets

Consider first the case in which trading is intermediated in non-transparent markets, that is, in standard competitive markets with no centralized clearing or centralized counterparty (such as an exchange).

Opacity   In non-trasparent markets, there is no centralized clearing, nor any centralized counterparty that sees all trades. Thus, the trades or position of each agent i, $(\mathbf{z}_+^i, z_-^i)$, are *not* observed by other agents.

Prices, budget constraints, and individual maximization   Long and short positions will in general be traded at a unitary price q^j, where the apex j denotes the type of the agent in the short position. Note that the price depends on the short agent's type j, as the type determines the agent's endowment which is public knowledge and affects her probability of default. Importantly though, the price is not a schedule contingent on overall trades of agent j, that is, does *not* depend on her portfolio, since it is not observed.

The maximization problem of agent i in non-transparent markets is thus given by:

$$\max_{x_0^i, x_1^i, \mathbf{z}_+^i, z_-^i} u^i\left(x_0^i\right) + \sum_{s=1}^{S} p_s u^i\left(x_1^i(s)\right)$$

s.t.
$$x_0^i = w_0^i - \sum_j q^j z_+^{ij} + q^i z_-^i, \tag{8}$$
$$x_1^i(s) = \max\left\{w_1^i(s) + \sum_j R^j(s) z_+^{ij} - R(s) z_-^i, (1-\alpha) w_1^i(s) - \varepsilon z_-^i\right\}$$

where $z_+^{ij}, z_-^i \geq 0$, for any j.

Competitive equilibrium In equilibrium, financial markets clear:[19]

$$\sum_i z_+^{ij} - z_-^j = 0, \text{ for any } j. \tag{9}$$

Furthermore, the equilibrium payoffs $R^j(s)$ satisfy the condition:

$$R^j(s) = R^j(\mathbf{z}_+, \mathbf{z}_-; s). \tag{10}$$

Let

$$m^i(s) = MRS^i(s) \equiv p_s \frac{\frac{\partial u^i\left(x_0^i, x_1^i(s)\right)}{\partial x_1^i}}{\frac{\partial u^i\left(x_0^i, x_1^i(s)\right)}{\partial x_0^i}} \tag{11}$$

denote the marginal rate of substitution between date 0 and state s at date 1 for agents of type i at equilibrium; that is, the *stochastic discount factor* of agents of type i. The equilibrium price of an asset is then simply equal to the discounted value of asset payoffs, where the discount rate is adjusted for risk according to the stochastic discount factor of any agent with a long position in the asset. More precisely, agents with a long position in the asset are those who have the highest marginal valuation for the asset's return, and

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[19]For simplicity we state in the text equilibrium conditions for the case of symmetric equilibria, where all agents of type i take the same default and portfolio choices, for any i. The proof of existence of equilibria requires however that we allow for asymmetric equilibria, so as to exploit the presence of a continuum of agents of the same type to convexify their choices. This allows us to formally identify the payoff correspondence of the economy with the convex hull of $\mathbf{R}\left(\mathbf{z}_+, \mathbf{z}_-; s\right)$.

hence at equilibrium, prices q^j satisfy:

$$q^j = \max_i E\left(m^i R^j\right), \text{ for any } j, \text{[20]} \tag{12}$$

where R^j is the random variable whose realization in state s is $R^j(s)$.

3.2 Centralized clearing

In the previous section, we formalized the competitive equilibrium of an economy in which financial market trades are intermediated in non-trasparent markets.

Transparency   In this section we model instead the operation of a centralized clearing mechanism characterized by *position transparency*. Transparency is obtained because a centralized clearing mechanism is assumed to aggregate all the information about trades and disseminate it to market participants.[21]

Regarding bankruptcy resolution, we continue to assume that no creditor has direct privileged recourse to a debtor's collateral in case of default; and that, at equilibrium, the sum total of cash flows received by the debtor is distributed pro-rata among the holders of long positions against the debtor. As in opaque markets, the equilibrium payoff of the asset shorted by agent j is given by (7).

Prices, budget constraints, and individual maximization   Because of position transparency, each agent in the economy has access to detailed information about all trades and can condition contract terms on this information. In particular, the price an agent j will face for a short position on the asset will in general reflect her default decision as well as the expected payoff of the asset in case of default, state by state. The agent's default

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[20]Alternatively, but equivalently, the equilibrium price for any j can be written as follows: $q^j = E\left(m^i R^j\right)$, for any i s.t. $z_+^{ij} > 0$ and $q^j = \max_i E\left(m^i R^j\right)$, if $z_+^{ij} = 0$ for any i.

[21]Two points are in order. First, in the model, transparency provided by centralized clearing mechanism obtains coincidentally with the submission and execution of trades. Our equilibrium setup cannot deal with the timing or market micro-structure issues associated with *when* trades are submitted and *when* they are made transparent. We discuss this issue in some detail in Section 4.

decisions will depend on her portfolio position and on the expected payoff of her long positions, which in turn reflect all other agents' default decisions at equilibrium. This way, the price an agent j will face for a short position on the asset will depend at equilibrium on all agents' portfolio positions.

When shorting the asset, therefore, an agent j will choose her own portfolio of short and long positions in the asset, but she will also want to choose the counterparties to trade the asset with, that is, in effect, the portfolio positions of the counterparties. When hedging a short position, for instance through a corresponding long position, each agent j will face the following trade-off: buying a long positions with a highly leveraged agent will be cheaper but will in turn reduce the price he/she can obtain from his/her shorting, as all traders will realize the fragility of his/her hedging position.

We therefore need to model an agent choosing his/her own portfolio as well as the counterparties to trade the asset with. But each agent's counterparties are characterized by their own type and portfolios, and, in turn, by those of their conterparties. Formally, this is obtained by positing that each agent i chooses a portfolio vector for every type in the economy,[22] say $\mathbf{t}^i = \left(\mathbf{t}^{ij}\right)_{j \in I} \in \mathbb{R}^{I(I+1)}$, where $\mathbf{t}^{ii}$ is intended to represent the agent's own portfolio; that is, $\mathbf{t}^{ii} = \left(\mathbf{z}^i_+, z^i_-\right)$. One way to interpret this formulation is that the individual mazimization problem of each agent determines his/her demand of counterparties, where counterparties are characterized by their type, portfolios, the type and portfolios of their counterparties, and so on. The market-clearing notion we impose will then guarantee that each agent will trade with the counterparties characterized by the portfolio vectors he/she will choose. That is, at equilibrium the demand and the supply of counterparties match.

Prices in turn will reflect the richness of the strategy space we have adopted: an agent of type j with portfolio position $\left(\mathbf{z}^j_+, z^j_-\right)$ will face an ask price map

$$q^j\left(\mathbf{t}^j\right) = \max_i E\left(m^i R^j\left(\mathbf{t}^j\right)\right). \tag{13}$$

That is, an agent of type j understands that the price it will face for a short position depends on the total short positions it sells, z^j_-. Furthermore, an agent of type j understands that the price it will face for a short position depends also on the payoff of its long portfolio $\mathbf{z}^j_+$, which in turn depends

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[22]There is a convenient redundancy in this formulation, of course: each agent's problem will only depend on the portfolio vector of his/her counterparties, which might not include all types in the economy.

indirectly on the portfolios of all the other agents in the economy, $R^i(\mathbf{t}^i)$, for all i.

In this context, therefore, different agents will face different prices, reflecting the probability of default implied by their characteristics: their type (e.g., level of endowment) as well as their and everybody else's trading positions. Nonetheless, we assume that prices are set in a competitive manner. Specifically, agents are price-takers. This requires us to conceive a non-standard formulation of the price-taking assumption for short positions (similar in spirit to Acharya and Bisin, 2008, and Bisin, Gottardi and Ruta, 2009). Specifically, an agent of type j understands that the price it will face for a short position will reflect a risk adjustment according to the stochastic discount factor of the agents who would hold such a short position, that is, of those agents who share the highest marginal valuation for the payoff associated to its position, $R^j(\mathbf{t}^j)$. Price taking is then represented by the fact that agents take the vector of stochastic discount factors $\left(m^1, \ldots, m^i, \ldots, m^I\right)$ as given. On the other hand, regarding long positions, the price q^j is taken as given by each agent.

The individual maximization problem of agent i is thus given by:

$$\max_{x_0^i, x_1^i, \mathbf{t}^i} u^i\left(x_0^i\right) + \sum_{s=1}^{S} p_s u^i\left(x_1^i(s)\right)$$

$s.t.$
$$x_0^i = w_0^i - \sum_j q^j(\mathbf{t}^i)z_+^{ij} + q^i(\mathbf{t}^i)z_-^i,$$
$$x_1^i(s) = \max\left\{w_1^i(s) + \sum_j R^j(\mathbf{t}^j; s)z_+^{ij} - R(s)z_-^i, (1-\alpha)w_1^i(s) - \varepsilon z_-^i\right\}$$

where $\mathbf{t}^{ij} \geq 0$, for any $i, j \in I$ and $\mathbf{t}^{ii} = \left(\mathbf{z}_+^i, z_-^i\right)$.

(14)

Competitive equilibrium At equilibrium, the positions agents choose for their conterparties must be consistent with the position the counterparties indeed choose for themselves:

$$\mathbf{t}^{ij} = \left(\mathbf{z}_+^j, z_-^j\right), \quad \text{for any } i, j \in I \tag{15}$$

all markets clear:

$$\sum_i z_+^{ij} - z_-^j = 0, \text{ for any } j, \tag{16}$$

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and the price maps are rationally anticipated by agents:[23]

$$q^j\left(\mathbf{t}^j\right) = \max_i E\left(m^i R^j\left(\mathbf{t}^j\right)\right).\qquad(17)$$

Modeling agents choosing positions for all other agents in the economy is non-standard. It is worth clarifying a few points in this regard. First of all, other agents' positions enter each agent's maximization problem by affecting the prices at which he/she can short the asset. This is because the positions of each agent's counterparties affect his/her probability of default as well as the expected repayment of the asset he/she shorted in the event of default. Of course each agent is indifferent with respect to the positions of agents who are not his/her counterparties. The equilibrium conditions guarantee that no two agents will ever choose different positions for a third agent, that is, prices will adjust so that this will never be the case.[24]

3.3 Welfare

How does the competitive equilibrium in non-transparent markets compare in terms of efficiency properties to the competitive equilibrium under centralized clearing with transparency? To answer this question, we write down the constrained Pareto efficient outcome as the solution to the following problem:[25]

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[23]Our definition of competitive price maps can be thought of as capturing the same consistency condition required by Perfect Nash equilibrium in strategic environments: every agent understands that the ask price she will face for any (possibly out-of-equilibrium) short position z_-^j will depend on the willingness to pay of agents on the long side of the market. In a competitive equilibrium, however, all deviations from equilibrium are necessarily "small," and hence such willingness to pay coincides with the highest marginal valuation at equilibrium.

[24]It should be noted however that (even though we do not account for this in the notation, for simplicity) equilibrium existence requires that, to guarantee convexity, we allow for asymmetric equilibria. In other words, it is possible that, at equilibrium, agents of the same type choose different portfolios. In this case, the equilibrium conditions require that the fraction of agents demanding a specific type-portfolio combination in their counterparties will match the fraction of agents offering that combination.

[25]Our definition of Pareto efficiency implicitly requires symmetry, in the sense that we require allocations of all individual agents of type i to be the same, for any type i. This is just for notational simplicity, consistently with our definition of competitive equilibrium. All results however apply for the general case, which allows for asymmetric equilibria to convexify default decisions.

$$\max_{(x_0^i, x_1^i, z_+^{ij}, z_-^i)_{i,j}} \sum_i \lambda^i E\left(u^i(x_0^i, x_1^i)\right) \tag{18}$$

subject to

$$\sum_i x_0^i - w_0^i = 0,$$

$$\sum_i x_1^i(s) - w_1^i(s) = 0, \text{ for any } s$$

$$x_1^i(s) = \max\left\{w_1^i(s) + \sum_j R^j(\mathbf{z}_+, \mathbf{z}_-; s)z_+^{ij} - R(s)z_-^i, (1-\alpha)\,w_1^i(s) - \varepsilon z_-^i\right\},$$

where $\mathbf{z}_+, \mathbf{z}_- \geq 0$, and λ^i is the Pareto weight associated to agents of type i.

This is the standard constrained efficiency problem for a general equilibrium economy once it is assumed that default is not controlled by the planner. The constraints in program (18) serve two purposes: (i) it restricts the planner's allocations to those that can be achieved with the limited financial instruments available in the economy; andm (ii) it accounts for the fact that each agent can choose to default or not, in each state s: consumption in default state s is $(1-\alpha)\,w_1^i(s) - \varepsilon z_-^i$, the non-pleadgeable fraction of endowment net of the deadweight costs.[26]

3.4 Results

We can derive the following results on the constrained efficiency of the economy with centralized clearing and transparency, in contrast to the (generic) constrained inefficiency of the economy with non-transparent markets.

Proposition 1. *Any competitive equilibrium of an economy with a centralized clearing mechanism is constrained Pareto optimal.*

The intuition for efficiency of the economy with centralized clearing and transparency is that each agent j that is short on the asset faces a price $q^j(\mathbf{t}^j) = \max_i E\left(m^i R^j(\mathbf{t}^j)\right)$ that is conditioned on her positions and on the

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[26]Formally, the constraint includes the incentive compatibility constraint for each agent's choice of default:

$$u^i(x_0^i, x_1^i(s)) \geq u^i(x_0^i, (1-\alpha)\,w_1^i(s) - \varepsilon z_-^i). \tag{19}$$

positions of the counterparties. In particular, any agent is allowed to choose the portfolio positions of the counterparties she trades with (and at equilibrium her counterparties will indeed choose such positions). Consequently, each agent j internalizes the effect of her choices on her future decisions regarding default and on the payoff of the asset she is shorting. The observability of all trades allows for such conditioning of prices and internalization of any externality that trading and default choices impose on other agents.

We show that the opacity of markets induces inefficiencies through the counterparty risk externality. More specifically, we shall show that equilibria of an economy with *non-transparent markets* are typically constrained inefficient. In other words, the transparency provided by centralized clearing mechanism is necessary for constrained efficiency.

Proposition 2. *Competitive equilibria of economies with a centralized clearing mechanism cannot be robustly supported as equilibria in economies with non-transparent markets.*[27] *More specifically, any competitive equilibrium of the economy with centralized clearing mechanism in which default occurs with positive probability cannot be supported in the economy with non-transparent markets.*

The intuition is that in non-transparent markets, each agent j that is short on the asset faces a price q^j that is *not* conditioned on her portfolio position $\left(\mathbf{z}_+^j, z_-^j\right)$, nor on her counterparties' positions. Consequently, she does not internalize the effect of her default on the payoff of the asset she is shorting. This is a *counterparty risk externality*.

Finally, let the leverage of agent j, L^j, be defined as the value of her short positions' contractual payoff (promised debt payment) divided by the value of her endowment (asset value).

$$L^j \equiv \frac{E(m^j R z_-^j)}{E(m^j w_1^j)}. \tag{20}$$

Then,

Proposition 3. *For deadweight costs ε that are small enough, competitive equilibria of economies with non-transparent markets are characterized by*

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[27]Formally, by robustly we mean: *for an open set of economies parametrized by agents' endowments and preferences.*

weakly greater (and robustly by strictly greater) leverage and default risk compared to equilibria of the same economy with a centralized clearing mechanism.

Since ask prices in economies with non-transparent markets do not penalize the short positions for their own incentives to default, agents have incentives to exceed the Pareto efficient short positions. Indeed, the proof of these main propositions in the Appendix shows that as long as (i) the underlying asset has some aggregate risk, its price will robustly carry a risk premium that is positive, and (ii) bankruptcy costs are not too high (ε is small), then agents with endowments in the aggregate risky states do not have adequate commitment to avoid default, or conversely, agents have an incentive to go excessively short. This increases the equilibrium default rate and leads to inefficient risk-sharing.[28] For efficient risk-sharing, it is in general necessary to be able to commit to future payoffs on financial assets, but in non-transparent markets, such commitment cannot be ensured through prices.

Our analysis makes it precise that it is the *opacity* that leads to ex ante inefficiency in terms of excessively large short positions or leverage. In equilibrium, agents anticipate the lowering of payoff on long positions due to counterparty risk and the price of insurance falls. However, this is not sufficient to preclude the insurers from selling large quantities of insurance and defaulting ex post, as the risk premiums they earn (which depend on the ratio of price to the payoff) remain unaffected. In fact, agents respond to the externality by buying more insurance, a kind of "run" on the insurers' endowment, but this response makes the inefficiency (due to excessive bankruptcy costs) in the model only worse.

3.4.1 Opacity and counterparty risk externality

Markets in centralized clearing economies are efficient, but they require prices which are explicitly sensitive with respect to any variable affecting default decisions: prices of short positions on the part of agent j depend also on the portfolios of any agent i whose assets are in the portfolio of agent j. In

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[28]If $\varepsilon = 0$, z_{-}^i is unbounded and, strictly speaking, the economy has no equilibrium. This is just an extreme case, which is of interest to identify the "force" towards borrowing and default built into our model of non-transparent markets. Positive deadweight costs, $\varepsilon > 0$, guarantee the existence of equilibrium.

general, prices which only depend on the portfolio of the agent shorting the asset do not guarantee constrained efficiency of equilibrium.

More specifically, consider an economy with a centralized clearing mechanism but with prices which only depend on the portfolio of the agent shorting the asset. In this economy, at a competitive equilibrium, assets payoffs are consistent:

$$R^j(s) = \begin{cases} \frac{\sum_i R^i(s)(\mathbf{z}_+,\mathbf{z}_-)z^{ji} + \alpha w_1^j(s)}{z_-^j} & \text{if } Y_{ND}^j(\mathbf{z}_+,\mathbf{z}_-;s) \leq Y_D^j(\mathbf{z}_+,\mathbf{z}_-;s) \\ R(s) & \text{if } Y_{ND}^j(\mathbf{z}_+,\mathbf{z}_-;s) \leq Y_D^j(\mathbf{z}_+,\mathbf{z}_-;s) \end{cases} \tag{21}$$

and the price maps are rationally anticipated by agents:

$$q^j = q^j\left(\mathbf{z}_+^i, z_-^i\right) = \max_i E\left(m^i R^j\right). \tag{22}$$

It is however straightforward to prove the following.

Proposition 4. *Competitive equilibria of economies with a centralized clearing mechanism cannot be robustly supported as equilibria in economies with a centralized clearing mechanism but with prices which only depend on the portfolio of the agent shorting the asset.*

On the other hand, however, many economies of interest, like the one in the example in Section 2 will satisfy without loss of generality the condition that agents are only on one-side of the markets; that is,

$$z_+^{ij} z_-^i = 0, \text{ for any } i, j.$$

In this case, competitive equilibria of economies with a centralized clearing mechanism but with prices which only depend on the portfolio of the agent shorting the asset are indeed constrained efficient.

3.4.2 Netting

In the model in the previous section, an agent i is allowed to go both short and long on the asset, and in equilibrium it might be that $z_-^i > 0$ and, at the same time, $z_+^{ij} > 0$ with some counterparty j. It might even be the case that an agent i has simultaneously both long and short position with counterparty j on the asset: $z_+^{ij} > 0$ and $z_+^{ji} > 0$. These positions are generally not

redundant, as the return paid by the asset depends in equilibrium on the state-contingent default strategy of the shorting party.

Financial markets, however, often have in place various institutional mechanisms designed to reduce exposure to the insolvency risk of a counterparty, e.g., to net bilateral (and at times, multilateral) positions of opposite sign.[29] Netting introduces an asymmetry in treatment of long positions in the event of default: it allows some creditors direct privileged recourse to a debtor's collateral in case of default. As a consequence, at equilibrium, with netting, the sum total of cash flows received by the debtor would not be distributed pro-rata among the holders of long positions against the debtor; see Bergman, Bliss, Johnson, and Kaufman (2003) and Pirrong (2009) for detailed institutional analyses of netting.

In the context of our economy netting would be represented by payoffs at equilibrium on the asset shorted by agent j which depend on who holds the long position. Let the return of such trading positions in state s be denoted $R^{ij}(\mathbf{t}; s)$. If $z^{ij}_+ - z^{ji}_+ > 0$, so that, after netting, j is still a debtor with respect to i, then i is paid in full only on part of her long positions. The asset's ex-post return structure, for any possible trade positions, $R^{ij}(\mathbf{t}, s)$ takes in this case a complicated algebraic form, but all our analysis could be reproduced essentially with no modifications in the case of netting.

Proposition 5. *Competitive equilibria of economies with a centralized clearing mechanism and netting are constrained Pareto efficient and cannot be robustly supported as equilibria in economies with non-transparent markets and netting.*

In general, however, economies with and without netting cannot be ranked in terms of welfare. The introduction of netting changes the default mechanism ex post and hence it changes both the default decision ex ante as well as, indirectly, the equilibrium payoff of financial markets in the economy, possibly inducing distributional effects across agents which prevent welfare comparisons in terms of Pareto rankings.

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[29]Since the early 1990s, in particular, netting is allowed also in the accounting regulations adopted to determine a financial institution's capital ratio under the Basel II Accord.

3.5 Decentralization of centralized clearing

Our model highlights that the crucial aspect of centralized clearing and transparency is that agents can condition the terms of the contracts they trade on the total financial position of the counterparty and not just on bilateral positions. This is by construction a decentralized trading mechanism, though reduced-form, in the context of competitive equilibrium modeling we adopted. The issue of its implementation in actual financial markets remains open, especially considering that financial positions in practice are contracted sequentially.

What is required to implement competitive pricing and centralized clearing with transparency is a trading mechanism that allows prices and other contractual terms to adjust *continuously* with each agent's total position. Such a mechanism could look much like a margin or collateral arrangement. However, currently such arrangements are based on mark-to-market valuation of positions and an overall assessment of counterparty risk (e.g., through a credit rating). Hence, they are not exactly equivalent to continuously observing each agent's total position and conditioning price on that information. Such arrangements cannot preclude institutions from positions beyond a certain size, that is, cannot implement non-linear pricing schedules - or "position limits" - as often employed on clearinghouses and exchanges.

To allow conditioning of trades on overall positions of a counterparty, post-trade transparency - in which trades are conducted during the day, reconciled and registered with a centralized clearing agency at the end of the day, and transparency provided to market participants on these trades thereafter - is necessary. However, if economic behavior of institutions is not stationary, then post-trade transparency will in general not be sufficient for efficiency, and even pre-trade transparency may be necessary. This is because in absence of information about trades that an institution plans to undertake, it is not possible for counterparties to charge an appropriate pricing schedule. Such pre-trade transparency may be perceived to be relatively intrusive in some markets. We discuss several alternative mechanisms in the next section.

4 Regulatory mechanisms

Our model highlights that the crucial aspect of centralized clearing and transparency is that agents can condition the terms of the contracts they trade

on the total financial position of counterparties and not just on bilateral positions. This is a natural reduced-form trading mechanism in the context of competitive equilibrium modeling we adopted. What is required to implement competitive pricing and centralized clearing with transparency is a trading mechanism that allows prices and other contractual terms to adjust *continuously* with each agent's total position. Such a mechanism could look much like a margin or collateral arrangement.[30]

Other market regulation mechanism are or could be commonly employed to adress the counterparty risk externality. Our general set-up allows us to study their welfare properties. We discuss here in some detail two of them, namely bilateral collateral constraints and a bankruptcy rule imposing seniority of centrally cleared positions over non-transparent positions. Cash collatoral is common practice: IMF (April 2010) shows that the top five banks and broker dealers in the United States posted cash collateral on derivatives positions as of 1 December 2009, ranging from 15% of derivatives payables (in case of Goldman Sachs) to 50% (for Bank of America). Furthermore, some recent changes in OTC markets, especially in contract terms of standardized credit default swaps (the so-called "Big Bang" protocol laid out in April 2009), require counterparties to exchange a part of their exchanged risk in pre-funded terms, which are effectively collateral requirements. The subordination of non-transparent positions in bankruptcy has also been proposed as a possible regulatory tool in discussions at the International Monetary Fund and Financial Stability Board (Basel) for containing contingent risks linked to derivatives.

4.1 Collateral constraints

Consider our example economy of Section 2 in which selling one unit of the asset short requires posting k units of the date-0 commodity as "collateral" to the counterparty. We assume that, when posted as collateral, one unit of the date-0 commodity pays an exogenous constant return r. To start with, we will assume r is equal to one. The collateral is "segregated" for each counterparty in that it has privileged access to its collateral in case of default on the contract.

Then, agents of type 3 do not default in state B provided $w^3(B) + kz^3 -$

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[30]Currently, however, such arrangements are based on mark-to-market valuation of positions and only on an overall assessment of counterparty risk (e.g., through a credit rating).

$Rz^3 \geq -\varepsilon z^3$, which can be expressed as

$$kz^3 \geq Rz^3 - \varepsilon z^3 - w^3(B) \ , \tag{23}$$

a condition that provides a lower bound on the required collateral constraint to deter default. However, not all collateral constraints are feasible for posting by agents of type 3 at date 0. This date-0 budget constraint is

$$w_0^3 + qz^3 \geq kz^3 \ , \tag{24}$$

which yields an upper bound on the feasible collateral constraint.

Since in our example economy, efficiency is achieved when there is no default and $Rz_{ND}^3 = w^3(B)$, efficiency can be attained with a collateral constraint if and only if the lower bound in (23) is smaller than available resources for $z = z_{ND}^3$ which impose the upper bound in (24):

$$(R - \varepsilon)z_{ND}^3 - w^3(B) \leq w_0^3 + q_{ND}z_{ND}^3, \tag{25}$$

where q_{ND} is the equilibrium price of insurance absent any default.

It follows then that in this example, collateral constraints can in general achieve efficiency if and only if ε, the deadweight cost of bankruptcy for the insurer, is not too small. Intuitively, collateral adds to the insurer's liability from default since it is seized by the counterparty in case of insurer's default, but it is released for the insurer otherwise. The greater this liability, the lower are the insurer's ex-post incentives to default.[31] However, the insurer's ability to post collateral is limited by the starting endowment. When ε is small, the incentive to default is rather strong, so that counteracting it requires the insurer to post high levels of collateral; such high levels might, however, not be feasible given insurer's limited endowment.

Indeed, when ε is too small, collateral constraint k that rules out default needs to be so large that it restricts z^3, as given by $z^3 \leq \frac{w_0^3}{(k-q)}$, to a level that is smaller than z_{ND}^3, limiting the extent of risk-sharing in the economy to below efficient levels (even though insurer's default is averted). The supply of hedging by the insurer, z^3, is now decreasing in the extent of collateral constraint, k, whereas the price of insurance, q, is rising in k (but at a rate that is smaller than one). The insurers are rendered funding-constrained in a

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[31]This is unlike the traditional view of collateral which is to simply alleviate the limited pledgeability of endowments or to serve as a signaling device when there is asymmetric information about agents' endowments.

bid to avoid their default but this restricts equilibrium provision of insurance to inefficient levels.

Consider next the general economy with non-tranparent markets in which selling one unit of the asset short requires posting k units of the date-0 commodity as "collateral" to the counterparty, and the return on collateral (r) assumed to be equal to one.

An agent of type i with a short position $z_-^i > 0$ will default in state s iff:

$$w_1^i(s) + \sum_j R^j(s)z_+^{ij} - R(s)z_-^i < (1-\alpha)\, w_1^i(s) - (\varepsilon + k)\, z_-^i \,, \qquad (26)$$

confirming that the collateral constraints affect the default choice analogously to how the bankruptcy cost ε does. Nonetheless, controlling k, or even a type-dependent collateral constraint k^i, is *not* enough to induce optimal default.

To see this, recall that the return of the asset shorted by agent j depends on the entire set of portfolio positions in the economy, $(\mathbf{z}_+, \mathbf{z}_-)$. Efficiency requires in general that any agent j internalizes the effects of her positions and of the positions of her counterparts on her own default decisions. Efficiency would require therefore collateral constraints that depend on $(\mathbf{z}_+, \mathbf{z}_-)$. But collateral constraints of the form $(\mathbf{z}_+, \mathbf{z}_-)$ require the observability of $\left(\mathbf{z}_+^i, z_-^i\right)$, that is, a centralized clearing mechanism on the part of the regulator imposing the constraints, or the transparency of overall positions to counterparties. By implication, bilateral collateral constraints do not suffice in non-transparent markets to achieve efficiency of allocations.

Furthermore, as we have shown in the example, collateral constraints require agents to hold large quantities of collateral asset. We argue that even in the case where holding such large quantities of collateral asset is feasible, it might not in general be possible to obtain efficient allocations if the return on the collateral asset r is not adequately large. The key observation is that collateral constraints can now impose a mis-allocation cost on the economy as some agents are required to hold sub-optimal asset portfolios, specifically, a position in an asset that induces excessive consumption at date 1 for those agents who are shorting the asset.

Formally, let $z_-^{i*} > 0$ denote the efficient portfolio allocation of an agent i shorting the asset, and $(x_0^{i*}, x_1^{i*}(s))$ her consumption allocations. Let also k^* denote the (minimal) collateral constraint which guarantees that agent i, has no incentive to default in the collateral constraint economy, when she holds the optimal portfolio z_-^{i*}. The budget constraints of agent i, in the collateral constraint economy k^*, are

$$x_0^i - (q - k^*)z_-^i = w_0^i \ ,$$

$$x_1^i(s) = \max \left\{ w_1^i(s) + \sum_j R^j(s)z_+^{ij} - R(s)z_-^i + rk^*z^i, (1 - \alpha)w_1^i(s) \right\}.$$

It is clear then that any optimal allocation such that $z_-^{i*} > 0$ (and hence $x_0^{i*} > w_0^i$) can be decentralized with collateral constraints only if at equilibrium $q > k^*$, which does not necessarily have to hold. If instead $q \leq k^*$ the agent is constrained to consume an amount smaller or equal to her endowment at date 0.[32] Consider the case in which at equilibrium $q > k^*$. In this case, any allocation $x_0^{i*} > w_0^i$ can in fact be decentralized with collateral constraints, by choosing

$$z_-^i = \frac{q}{q - k} z_-^{i*} \ .$$

However, consumption at date 1 is now not optimal, unless $r = \frac{R(s)}{q}$. If $r < \frac{R(s)}{q}$, for all s, the collateral constraint is costly in terms of efficiency in that it requires agent i to hold an asset whose return is dominated. Note that if the collateral storage technology is *not* dominated, that is, if $\frac{R(s)}{q} < 1$, for some s, then the storage technology is a new asset in the economy and welfare comparisons are not meaningful, unless we introduce storage also in the baseline economy.

4.2 Bankruptcy design

A regulatory mechanism penalizing access into non-transparent markets might be welfare improving. One such regulatory mechanism is a bankruptcy rule imposing seniority of centrally cleared positions over non-transparent positions, that is, positions which are not reported to the centralized clearing mechanism. It would seem that with such subordination, junior non-transparent positions would not dilute the senior centrally cleared positions, for which counterparties would face appropriate incentives and risk controls.[33] This is however not the case in general.

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[32]When $q < k^*$, the optimal portfolio might even be infeasible for the agent; that is, $(k^* - q)z_-^{i*} > w_0^i$.

[33]Similar mechanisms are common in civil law countries, in the form of seniority rules favoring (transparent) notarized transactions over (opaque) bilateral ones. Thanks to Sabino Patruno for pointing this out.

Formally, consider our general economy with a centralized clearing mechanism. An agent i can trade a long position of an asset with nominal payoff $R(s)$ in state s, with counterparty j at price q^j; agent j, in turn, faces a price schedule $q^j(\mathbf{t}^j)$. Suppose now that the same asset can also be traded by agent j in non-transparent markets, and let $\left(z_+^{j,nt}, z_-^{j,nt}\right)$ denote her long and short positions in this market, respectively. The price agent j faces in non-transparent markets will depend on the agent's trading position in the centralized mechanism, which is transparently observable, but not depend on her position in non-transparent markets. That is, the price of the short position in non-transparent markets can be denoted as $q^{j,nt}(\mathbf{t}^j)$. The default decision of agent j will, however, depend on the entire set of positions in centralized and non-transparent markets, $\mathbf{z}_+, \mathbf{z}_-, \left(z_+^{j,nt}, z_-^{j,nt}\right)_{j\in J}$.

Of course, at equilibrium, agents trading long positions in non-transparent markets will take into account of their counterparties' incentives to default. As a consequence, the equilibrium price in non-transparent markets will account for the equilibrium default rate of short positions. Not surprisingly, as in our model with only non-transparent markets, this is cause for inefficiency: a counterparty risk externality exists within the non-transparent market. It is important, however, to understand whether this externality extends to the centralized clearing markets. That is, does a non-transparent market alongside a centralized clearing market have a negative externality on an otherwise efficient market mechanism, even if bankruptcy law guarantees the seniority of trades in the centralized clearing market?

The answer to this question depends on the properties of the bankruptcy institution. Even if the bankruptcy rule imposes seniority of centrally cleared positions over non-transparent positions, it is in fact still possible that trades in a non-transparent market create a negative externality on trades in a centralized clearing market. Consider as an illustration an economy with only a centralized clearing market in which an agent j who shorts the asset, at equilibrium, does not default in state s.

Consider the case, in particular, in which at equilibrium agent j shorts the asset as much as possible, without defaulting, that is, she is indifferent with regards to defaulting. The return in state s for agents with long positions against agent j is then $R(s)$, if j does not default, and $\frac{\sum_j R^i(\mathbf{z}_+, \mathbf{z}_-; s) z_+^{ji} + \alpha w_1^j(s)}{z_-^j} = R(s) - \varepsilon$, if agent j defaults. Any larger short position of agent j would induce her to default. Suppose now we allow agents to trade also in non-transparent markets and suppose that agent j indeed trades there to increase

her short position. If agent j decides to default in state s on her positions in the non-transparent market and bankruptcy is called, her counterparties in the centralized clearing market might obtain returns equal to $R(s) - \varepsilon$, or even smaller, depending on agent j's position in the non-transparent market. In this case, the opening of non-transparent markets, even if junior in bankruptcy with respect to the centralized clearing market, would impose a negative externality on the latter.[34]

But suppose instead the bankruptcy institution is such that agents are allowed to default selectively in the non-transparent market and not in the centralized clearing market. Consider in particular a bankruptcy institution as follows: i) if agent j defaults on her positions on counterparties in centralized markets only those positions are dealt in bankruptcy court and bankruptcy costs are εz_-^j, that is, independent of short positions in non-transparent markets; and, ii) if she does not default on her positions on counterparties in centralized markets, then counterparties in non-transparent markets can call bankruptcy and get access to j's collateral in excess of what is paid to positions in centralized markets (that is, in excess of $(1-\alpha)w_1^j$); in this case bankruptcy costs amount to $\varepsilon z_-^{j,nt}$.

Note that under this bankruptcy institution, no bankruptcy costs would ensue when an agent pays in full all her obligations with respect to counterparties in centralized clearing markets but does not pay counterparties in non-transparent markets and there is not enough of the agent's collateral for them to recover any payment in bankruptcy. This is because a creditor in a non-transparent market would not obtain any repayment either way (bankruptcy or not).[35]

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[34]This is as long as agent j will have an incentive to trade in the non-transparent market. This is in fact the case if the price she obtains for a short position in that market, $q^{j,nt}\left(\mathbf{t}_+^j\right)$, is greater than ε, which occurs robustly. Note that $q^{j,nt}\left(\mathbf{t}_+^j\right)$ is not the equilibrium price of the economy with both centralized clearing markets and non-transparent markets. It is rather the price at the equilibrium of the economy with no non-transparent markets, that is, the maximal marginal valuation across counterparties of agent j's short position in non-transparent markets.

[35]With respect to the economy with only a centralized clearing market, allowing for trades in non-transparent markets would allow for greater default and this can effectively open up the set of possible state-contingent trades in the economy. Furthermore, non-transparent markets might induce efficiency gain due to the creation of non-standardized financial products, a feature not accounted for in our model.

5 Conclusions and policy implications

In this paper, we formalized an important market failure arising due to opacity of markets, in particular that the payoff on each position depends in default on other positions sold by the defaulting party, but without position transparency, there is no way for market participants to condition their trades or prices based on knowledge of these other positions. We showed that this counterparty risk externality can lead to excessive default, and more generally, to inefficient risk-sharing. Centralized clearing, by enabling transparency of trades, and exchanges, by creating a centralized counterparty to all trades, can help agents fully internalize the counterparty risk externality. The model also helps evaluate effectiveness of proposed remedies for limiting this excess such as position-level transparency, centralized clearing or counterparty, collateral requirements, and subordination of OTC claims relative to centrally cleared ones.

We interpret our model as providing one explanation – based on incentives to excessively sell short, collect risk premiums and default ex post – for the substantial buildup of OTC positions in credit default swaps in the period leading up to the crisis of 2007-09. Our theoretical analysis can then help provide a normative framework for evaluation of several recent proposals for reform in for OTC markets.

Acharya, Engle, Figlewski, Lynch and Subrahmanyam (2009) divide the reform proposals into requiring a (i) centralized registry with no disclosure to market participants; (ii) centralized clearing with disclosure of aggregate trade information to market participants; and (iii) centralized counterparty or exchange with full public disclosure of prices and volumes. Our theoretical analysis makes it clear that a centralized registry by itself is *not* sufficient as it only gives regulators ex-post access to trade-level information but does not counteract the ex-ante moral hazard of institutions wanting to take on excessive leverage. Both centralized clearing and exchange improve on this ground but it is position transparency that is crucial. In our model, it is sufficient that centralized clearing can disseminate trade positions to market participants who then themselves set price schedules and risk controls conditional on that information. In particular, requiring all trades to take place through a centralized exchange is not necessary though in that case there would be no need to disclose information on all trades to individual agents.

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Appendix 1: Proofs

Note: Consider the following representation of preferences:

$$u_1^i(x) = E(x - f(x)) + \alpha f(Ex)$$

This representation reduces to expected utility for $\alpha = 0$ and to the specification of mean variance in Section 2 for $\alpha = 1$ and $f(x) = (\gamma/2)x^2$. All results in the paper would go through in this case.

Proof of Lemma 1. Let $\mathbf{R}(\mathbf{z}_+, \mathbf{z}_-; s)$ be the vector map of payoffs in state s obtained by stacking $R^j(\mathbf{z}_+, \mathbf{z}_-; s)$ for any $j \in I$.

For any $(\mathbf{z}_+, \mathbf{z}_-) \in \mathbb{R}_+^{I(I+1)}$, equation (7) defines a map from $\mathbb{R}_+^{SI}$ into $\mathbb{R}_+^{SI}$. In fact, for given $(\mathbf{z}_+, \mathbf{z}_-)$, payoffs are without loss of generality restricted to the compact set $[0, R(s)]$. Furthermore, by appropriately randomizing over ties, the map defined by equation (7) can be represented by a convex-valued map, the convex hull of $\mathbf{R}(\mathbf{z}_+, \mathbf{z}_-; s)$, for given $(\mathbf{z}_+, \mathbf{z}_-)$. It is then an upper-hemi-continuous convex-valued self-map on a compact set and Kakutani Fixed Point theorem guarantees existence of a fixed point. Furthermore, the map defined by (7) is upper-hemi-continuous in $(\mathbf{z}_+, \mathbf{z}_-)$. This can be shown directly from the definition of upper-hemi-continuity for correspondeces by means of a limit argument. As a consequence, $\mathbf{R}(s)(\mathbf{z}_+, \mathbf{z}_-)$, as a correspondence from $\mathbb{R}_+^{I(I+1)}$ into $\mathbb{R}_+^{SI}$ is indeed non-empty-valued and upper-hemi-continuous. ∎

As noted in the text, the proof of existence of equilibria requires that we allow for asymmetric equilibria, so as to exploit the presence of a continuum of agents of the same type to convexify their default choices. A standard argument allows then to identify the payoff correspondence of the economy with the convex hull of $\mathbf{R}(\mathbf{z}_+, \mathbf{z}_-; s)$. Under standard conditions on preferences the economy is then convex and the existence of (possibly asymmetric) competitive equilibria holds. The issue of satiation for general preference formulations can be dealt with via standard methods in the literature. See Bisin, Gottardi, and Ruta (2011) for details in a incomplete market economy with production which turns out to be a related environment.

Proof of Proposition 1. Let $(\mathbf{x}_0, \mathbf{x}_1) = (\mathbf{x}_0^i, \mathbf{x}_1^i)_{i \in I}$ denote the competitive equilibrium consumption allocation of an economy with a centralized clearing mechanism. Let $(\mathbf{z}_+, \mathbf{z}_-)$ be the portfolio allocation at equilibrium and $\mathbf{q}(\mathbf{z}_+, \mathbf{z}_-) = (q^i(\mathbf{z}_+, \mathbf{z}_-))_{i \in I}$ the price vector.

The constrained efficiency of such an allocation can be established by an argument essentially analogous to the one used to establish the Pareto optimality of competitive equilibria in Arrow-Debreu economies (also, issues of satiation for general preference formulations can be dealt with via standard methods in the literature). The proof proceeds by contradiction. Consider another allocation which, by assumption, Pareto dominates the equilibrium allocation $(\mathbf{x}_0, \mathbf{x}_1, \mathbf{z}_+, \mathbf{z}_-)$; let it be denoted $(\widehat{\mathbf{x}}_0, \widehat{\mathbf{x}}_1, \widehat{\mathbf{z}}_+, \widehat{\mathbf{z}}_-)$. By assumption it also satisfies the constraints

$$\widehat{x}_1^i(s) = \max\left\{w_1^i(s) + \sum_j R^j(\widehat{\mathbf{z}}_+, \widehat{\mathbf{z}}_-; s)\widehat{z}_+^{ij} - R(s)\widehat{z}_-^i, (1-\alpha)\, w_1^i(s) - \varepsilon\widehat{z}_-^i\right\}$$

in the definition of Pareto efficiency.

The crucial step in the proof requires noticing that such an allocation is available for any agent in the competitive markets of the economy with with a centralized clearing mechanism. In particular, consumption at time 1, for any agent i would be supported by portfolio $(\widehat{\mathbf{z}}_+, \widehat{\mathbf{z}}_-)$ at prices $\mathbf{q}(\widehat{\mathbf{z}}_+, \widehat{\mathbf{z}}_-)$. Recall in fact that each agent i chooses a whole vector $\mathbf{t}^i \in \mathbb{R}_+^{I(I+1)}$. As a consequence, if at equilibrium agents choose $(\mathbf{x}_0, \mathbf{x}_1, \mathbf{z}_+, \mathbf{z}_-)$ it must be that the allocation $(\widehat{\mathbf{x}}_0, \widehat{\mathbf{x}}_1, \widehat{\mathbf{z}}_+, \widehat{\mathbf{z}}_-)$ is not budget feasible with respect to the budget constraint at time 0,

$$\widehat{x}_0^i \geq w_0^i - \sum_j q^j(\widehat{\mathbf{z}}_+, \widehat{\mathbf{z}}_-)\, \widehat{z}_+^{ij} + q^i(\widehat{\mathbf{z}}_+, \widehat{\mathbf{z}}_-)\, \widehat{z}_-^i$$

with $>$ for at least one agent i. As in the standard proof of Pareto optimality of competitive equilibria in Arrow-Debreu economies, this implies that the allocation $(\widehat{\mathbf{x}}_0, \widehat{\mathbf{x}}_1, \widehat{\mathbf{z}}_+, \widehat{\mathbf{z}}_-)$ is not feasible; the desired contradiction. ∎

Proof of Proposition 2. We restrict attention to the case when ε is small so that there is default in the economy. Furthermore, assume to start with that $\varepsilon = 0$. Let $(\mathbf{z}_+^i, z_-^i)$ be the equilibrium portfolio for agent i in an economy with a centralized clearing mechanism. Let $S(i) \subseteq S$ denote the subset of the states of uncertainty in which, at equilibrium, an agent i will default. Then, $S(i)$ is robustly non-empty. Furthermore, if $S(i)$ is non-empty, then $z_-^i > 0$. For any economy such that $S(i)$ is non-empty (and $z_-^i > 0$) for some i, at equilibrium of the centralized clearing mechanism, we must have

$$q^i(\mathbf{z}_+, \mathbf{z}_-) = \sum_{s \in S(i)} p_s m^i(s) \frac{\sum_j R^j(\mathbf{z}_+, \mathbf{z}_-; s)\, z_+^{ij} + \alpha w^i(s)}{z_-^i} + \sum_{s \in S \setminus S(i)} p_s m^i(s) R(s).$$

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Suppose, by contradiction, that such a competitive equilibrium of the centralized exchange economy can be supported in an economy with non-transparent markets. Then it is necessarily supported by price $q^i = \sum\limits_{s \in S(i)} p_s m^i(s) \frac{\sum_j R^j(s) z_+^{ij} + \alpha w^i(s)}{z_-^i} + \sum\limits_{s \in S \setminus S(i)} p_s m^i(s) R(s)$, such that $R^j(s) = R^j(\mathbf{z}_+, \mathbf{z}_-; s)$, and hence $q^i = q^i(\mathbf{z}_+, \mathbf{z}_-)$, at the equilibrium portfolio. It is straightforward to see that in this case, at price q^i agent i prefers a portfolio $(\mathbf{z}_+^i, z_-^i + dz)$, for some $dz > 0$. This is because the marginal valuation of the discounted repayment of a unitary extra short portfolio dz, $\sum\limits_{s \in S(i)} p_s m^i(s) \frac{\sum_j R^j(s) z_+^{ij} + \alpha w^i(s)}{z_-^i} + \sum\limits_{s \in S \setminus S(i)} p_s m^i(s) R(s)$, depends negatively on z_-^i; while the price obtained at time 0 from the same unitary extra short portfolio, dz, q^i, does not. Since the portfolio $(\mathbf{z}_+^i, z_-^i + dz)$ is budget feasible, a contradiction is reached. This is the case for any equilibrium of the centralized exchange economy such that $S(i)$ is non-empty, for some i, and hence the contradiction holds robustly. The proof extends by continuity to ε sufficiently small. $\blacksquare$

Proof of Proposition 3. Once again, assume $\varepsilon = 0$ and the proof below extends by continuity to ε sufficiently small. Finally, the "weakly greater" part of the statement is straightforward. We turn to prove the robustly "strictly greater" part.

Consider the robust subset of economies for which, with centralized clearing at equilibrium, $S(i)$ is non-empty. An argument analogous to the one in the proof of Proposition 2 guarantees that, for these economies, when ε is small enough, at an equilibrium of the economy with non-transparent markets, $S(i) = S$. Agents i, in other words, default in all states $s \in S$. This proves that default is robustly strictly greater at equilibria of the economy with non-transparent markets than with centralized clearing.

Consider such an equilibrium with non-transparent markets, to study leverage. Consider now the general case in which $\varepsilon > 0$. At equilibrium it must be that $q^i > 0$. Suppose on the contrary that $q^i \leq 0$. In this case, we claim agents i would rather choose $z_-^i = 0$ and $z_+^{ij} > 0$ and hence would trivially not default. In fact, if $S(i) = S$, and $q^i \leq 0$, agents i would consume

$$x_0^i = w_0^i + q^i z_-^i - \sum_j q^j z_+^{ij}$$

$$x_1^i(s) = (1 - \alpha) w_1^i(s) - \varepsilon z_-^i,$$

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But then

$$
\begin{aligned}
x_0^i &= w_0^i + q^i z_-^i - \sum_j q^j z_+^{ij} \\
x_1^i(s) &= (1 - \alpha)\, w_1^i(s) - \varepsilon z_-^i
\end{aligned}
$$

By resorting to $z_-^i = 0$, instead agents i would guarantee themselves

$$
\begin{aligned}
x_0^i &= w_0^i - \sum_j q^j z_+^{ij} \\
x_1^i(s) &= w_1^i(s) + \sum_j R^j(s) z_+^{ij}
\end{aligned}
$$

which they prefer. Prices such that $q^i \leq 0$ therefore imply no default. This is the case for all agents of all types i. But then $R^j(s) = R(s)$, for all $s \in S$ and z_+^{ji} is robustly > 0, for some j, a contradiction with market clearing. At an equilibrium of the economy with non-transparent markets, therefore, it must be that $q^i > 0$. In this case z_-^i grows unbounded as $\varepsilon \to 0$. This proves that leverage is robustly strictly greater in the economy with non-transparent markets than with centralized exchange for ε small enough. ∎

Proof of Proposition 4. We shall only show here that the argument we used in the proof of Proposition 1 to prove the constrained efficiency of equilibrium allocations of economies with a centralized clearing mechanism breaks down when applied to economies with a centralized clearing mechanism but with prices which only depend on the portfolio of the agent shorting the asset, $\left(\mathbf{z}_+^i, z_-^i\right)$. A robust economy with constrained inefficient equilibrium allocations can be easily constructed; see Acharya and Bisin (2012).

Proceeding by contradiction. Consider another allocation, $(\widehat{\mathbf{x}}_0, \widehat{\mathbf{x}}_1, \widehat{\mathbf{z}}_+, \widehat{\mathbf{z}}_-)$, which Pareto dominates the equilibrium allocation and satisfies the constraints

$$
\widehat{x}_1^i(s) = \max\left\{ w_1^i(s) + \sum_j R^j(\widehat{\mathbf{z}}_+, \widehat{\mathbf{z}}_-; s)\widehat{z}_+^{ij} - R(s)\widehat{z}_-^i, (1 - \alpha)\, w_1^i(s) - \varepsilon\widehat{z}_-^i \right\}
$$

in the definition of Pareto efficiency.

This allocation might not be available for any agent in the competitive markets of the economy with with a centralized clearing mechanism but with prices which only depend on the portfolio of the agent shorting the

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asset. Indeed an agent i has only financial markets available with payoff $R^j(\mathbf{z}_+, \mathbf{z}_-)$, for any $j \in I \backslash i$, which she takes as given. As a consequence, we cannot conclude that the allocation $(\widehat{\mathbf{x}}_0, \widehat{\mathbf{x}}_1, \widehat{\mathbf{z}}_+, \widehat{\mathbf{z}}_-)$ is not budget feasible with respect to the budget constraint at time 0. ■

Proof of Proposition 5. The proof follows closely and straightforwardly the proofs of Proposition 1 and 2. It is therefore left to the reader. ■

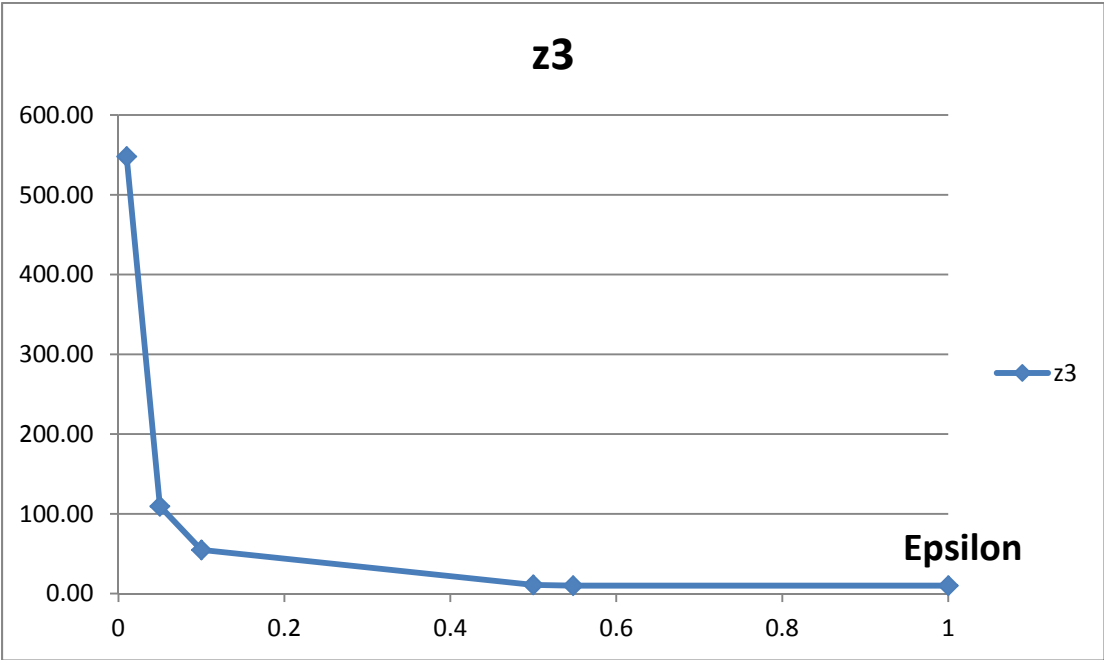Figure 1: The quantity of insurance sold (z^3) as a function of the deadweight cost of default (ε)

Figure 2: The realized payoff on the insurance (R^+) as a function of the deadweight cost of default (ε)
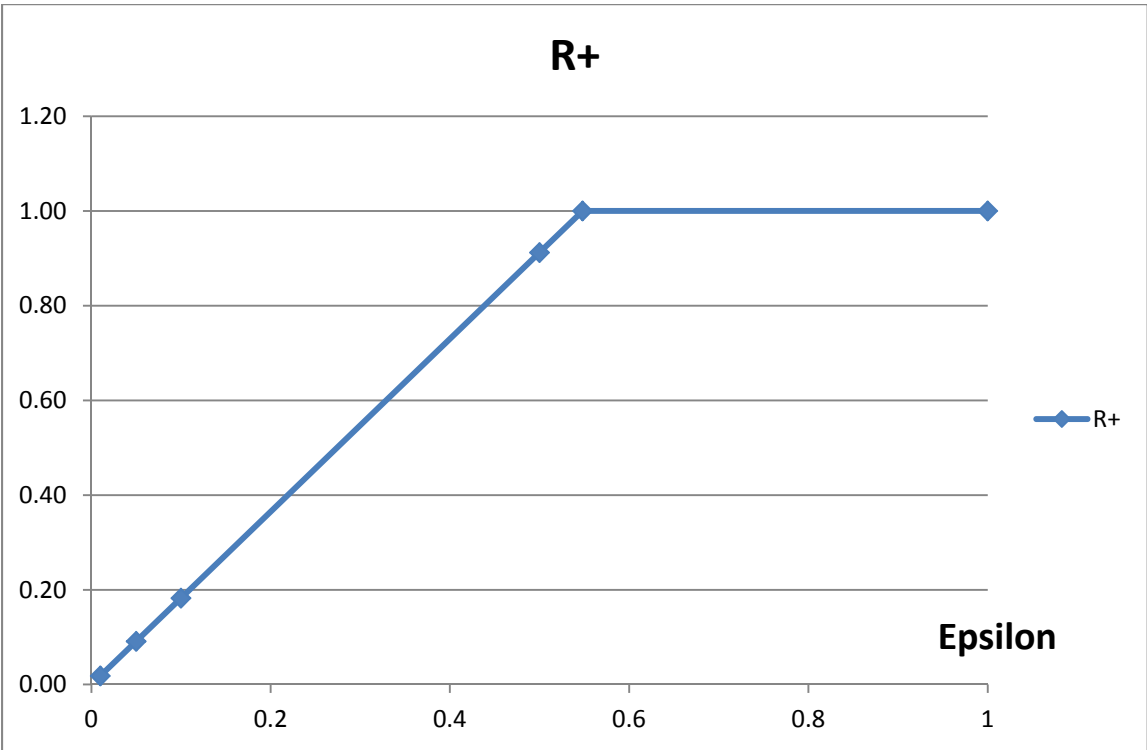
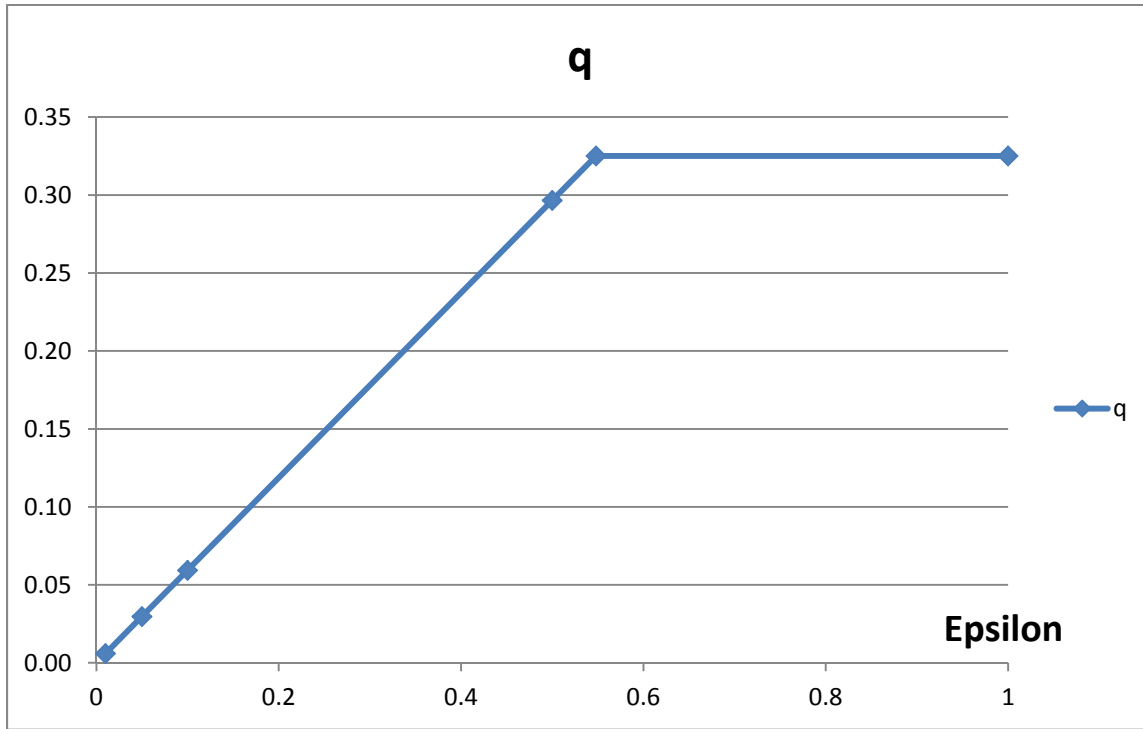Figure 3: The equilibrium price of insurance (q) as a function of the deadweight cost of default (ε)

Figure 4: The equilibrium utilities as a function of the deadweight cost of default (ε)